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Visualization and Processing of Tensors and Higher Order Descriptors for Multi-Valued Data

With 110 Figures, 85 in color

Springer

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Preface

This book presents a broad and illustrative sample of the state of the art of the emerging field of visualization and processing of tensor fields and higher-order descriptors. Topics range from applications of the analysis of tensor fields to research into their mathematical and analytical properties. Tensors arise in multiple areas of mathematics, science, and technology. For example, in physics of continuous media, tensor quantities arise in constructive equations that describe charge, mass, momentum, and energy transport (the diffusion tensor, the electrical conductivity tensor, etc.). In fact, most nontrivial presentations of theories of physics resort to tensor formulations. This is true for most of the theoretical branches, such as quantum and relativistic mechanics, thermodynamics, and electromagnetism. Hence, tensor analysis is a well-established branch of mathematics that is used extensively in many areas of science and engineering. Nevertheless, tensor formulations are typically not usual in image processing and related fields, even though, for many scenarios, they should be of major interest.

The motivation is that by collecting chapters from a range of fields (e.g., including medical imaging and scientific visualization), the field of tensor processing can be more clearly presented to the interested scientific community. The field itself may be cross-fertilized with innovations bridging the various research areas. We believe that this book contributes toward this goal by combining theoretical and experimental results, reflecting recent advances in tensor signal processing and opening new avenues of research.

This book consists of the following five parts.

Part I "Tensor Data Visualization" consists of two chapters. The first chapter gives an overview of techniques for visualization of tensors and tensor fields in engineering and discusses the current state of the art and challenges. Creating visualization tools for engineering tensors often involves solving multiple different technical problems at the same time, including visual intuitiveness, interactivity, and representation of uncertainty. The second chapter is about tensor invariants and glyph design and gives an overview of common glyphs, mostly with origins in mechanical engineering, and links their interpretation to specific tensor invariants.

Part II "Representation and Processing of Higher-Order Descriptors" consists of three chapters. The first chapter describes a matrix representation of local phase, a powerful concept that has been successfully used in many image processing applications. For multidimensional signals, the concept of phase is complex and there is no consensus on the precise meaning of phase. A remedy is suggested using a novel matrix representation of multidimensional phase. In the second chapter, extension of mathematical morphological operations techniques for matrix-valued images is used and extended to be used in vector images such as color images. In the third chapter, erosion is generalized to the space of diffusion-weighted MRI data. This is done effectively by solving a Hamilton-Jacobi-Bellman system (erosion) on the coupled space of three-dimensional positions and orientations, embedded as a quotient in the group of three-dimensional rigid body motions. The solution to the HJB equations is given by a well-posed morphological convolution.

Part III "Higher-Order Tensors and Riemannian-Finsler Geometry" includes four chapters that provide powerful mathematical language to model and analyze large and complex diffusion data such as high angular resolution diffusion imaging (HARDI) or diffusion kurtosis imaging (DKI). The first chapter gives a careful introduction to the foundations of higher-order tensor algebra and explains how some concepts from linear algebra generalize to the higher-order case. The second chapter discusses fourth-order symmetric tensors and how to model the positivity constraint present in diffusion. The third chapter describes the role of Riemann-Finsler geometry as a potentially powerful mathematical framework in the context of diffusion MRI and presents the basic theoretical foundation for Finsler-based tractography. The fourth chapter expands on these concepts and presents more details on a numerical Finsler tractography implementation.

Part IV "Tensor Signal Processing" presents new methods for processing of tensor-valued data. The first chapter gives a novel perspective on performing voxel-wise morphometry of diffusion tensor data using kernel-based approach. The second chapter reviews the free-water diffusion model and uses it to derive diffusion tensors following the elimination of the free-water component that is assumed to originate from the extracellular space. The third chapter reviews approaches that have been proposed to compute fabric tensors with emphasis on trabecular bone research. Fabric tensors aim to model both anisotropy and orientation of a material with respect to another one.

Part V "Applications of Tensor Processing" illustrates the use of tensors in different application domains. The first chapter reviews various types of tensors used in geometry processing and discusses the use of the metric and curvature tensors, two of the most studied tensors in geometry processing. The final two chapters demonstrate applications of diffusion-weighted imaging toward clinical use. One chapter describes a probabilistic technique for diagnostic prediction of first-episode schizophrenia patients based on their brain diffusion MRI data. The last chapter presents an interactive system that integrates the visual analysis of diffusion MRI tractography with data from electroencephalography (EEG).

We believe this collection of chapters captures the excitement and inspiration that has been generated during a series of Dagstuhl seminars devoted to visualization and

processing of tensor fields. This book includes contributions from attendees of the fourth meeting, entitled "Visualization and Processing of Tensors and Higher Order Descriptors for Multi-Valued Data," held in December 2011. As in the three earlier volumes, the authors report on recent research results and future directions for the analysis and visualization of tensor fields. One of the goals of this seminar is to bring together researchers from along the axis between pure and applied research, identifying new multidisciplinary research challenges. This book, we hope, will continue to further that goal in a broader context.

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Contents