Supplemental Material - A Fast Geometric Multigrid Method for Curved Surfaces

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1 EXTENDED COMPARISONS

In Table 1 (second page), we show more results for the Poisson problem on manifold meshes. Note that some entries are listed as NaN. This arises from the Gauss–Seidel smoothing step, where a divison by the diagonal of the system matrix is performed. In the cases where a NaN arises, the restriction of the system matrix results in zero-entries on the diagonal. This is not a fundamental issue for the Gauss–Seidel solver. The issue could, for example, be addressed by including a pivoting strategy. We did not include these entries for the conclusions listed in the main paper.

We also report a comparison with a data smoothing problem with $\alpha = 1 \times 10^{-3}$ for manifold meshes in Table 2 and for non-manifold meshes and point clouds in Table 3. Convergence plots for data smoothing on the manifold meshes in the main paper are shown in Figure 1 and Figure 2.

REFERENCES

Hsueh-Ti Derek Liu, Jiayi Eris Zhang, Mirela Ben-Chen, and Alec Jacobson. 2021. Surface Multigrid via Intrinsic Prolongation. *ACM Trans. Graph.* 40, 4 (2021).

Lin Shi, Yizhou Yu, Nathan Bell, and Wei-Wen Feng. 2006. A fast multigrid algorithm for mesh deformation. *ACM Trans. Graph.* 25, 3 (2006), 1108–1117. https://doi.org/10.1145/1141911.1142001

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Table 1: Comparison of our hierarchy construction and solver for a Poisson problem with $\eta = 1 \times 10^{-6}$ mass matrix coefficient and tolerance of 1×10^{-4}. The maximum number of iterations for iterative solvers is set to 100.

Model	#Vert	Gravo MG (Ours)			Liu et al. [2021]			Shi et al. [2006]			AMG-RS			AMG-SA			Eigen		Pardiso	
		Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Fact.	Subst.	Fact.	Subst.
Beetle	19k	0.01	28	0.06	0.55	18	0.05	0.01	33	0.09	0.03	100	0.29	0.03	100	0.20	0.02	0.00	0.06	0.00
Ogre	19k	0.02	6	0.02	0.54	7	0.03	0.01	13	0.04	0.03	100	0.27	0.04	40	0.08	0.02	0.00	0.06	0.00
Screwdriver	27k	0.02	4	0.03	0.75	4	0.03	0.01	9	0.04	0.03	100	0.35	0.05	24	0.07	0.05	0.00	0.09	0.00
Mumble	34k	0.03	10	0.05	0.97	6	0.04	0.02	9	0.04	0.04	100	0.41	0.05	57	0.17	0.04	0.00	0.11	0.01
Horse	48k	0.04	7	0.05	1.42	6	0.06	0.02	12	0.08	0.05	100	0.58	0.08	41	0.19	0.10	0.00	0.14	0.01
Laurent's Hand	50k	0.04	5	0.07	1.59	4	0.06	0.02	23	0.18	0.06	100	0.66	0.10	36	0.21	0.10	0.00	0.18	0.01
Dinosaur	56k	0.04	7	0.06	1.71	5	0.06	0.02	17	0.14	0.06	100	0.65	0.09	49	0.28	0.08	0.00	0.18	0.01
Heart	78k	0.06	15	0.18	2.90	29	0.38	0.04	33	0.43	0.08	100	1.06	0.13	100	0.95	0.17	0.01	0.27	0.01
Hannya Mask	83k	0.07	15	0.19	2.56	4	0.09	0.05	19	0.26	0.09	100	1.03	0.13	100	0.91	0.17	0.01	0.30	0.02
Trex	100k	0.10	9	0.16	3.40	4	0.14	0.05	42	0.64	0.12	100	1.39	0.22	97	1.10	0.18	0.01	0.38	0.02
The Thinker	110k	0.09	4	0.08	3.50	4	0.11	0.04	13	0.19	0.10	100	1.10	0.20	26	0.26	0.36	0.01	0.39	0.02
Egea	134k	0.11	4	0.13	4.42	4	0.14	0.06	15	0.31	0.16	100	1.79	0.23	27	0.36	0.94	0.02	0.50	0.03
Sappho's Head	140k	0.11	6	0.17	4.36	7	0.21	0.07	14	0.35	0.15	100	1.79	0.24	45	0.73	0.38	0.01	0.51	0.03
Human Torso	142k	0.15	4	0.15	4.97	5	0.20	0.06	40	0.85	0.16	100	1.86	0.25	44	0.72	0.73	0.02	0.57	0.03
Aim Dragon	152k	0.14	7	0.19	5.14	8	0.27	0.06	27	0.62	0.15	26	0.46	0.31	29	0.48	0.81	0.02	0.58	0.04
Armadillo	172k	0.16	9	0.28	5.97	7	0.29	0.08	27	0.76	0.20	100	2.37	0.36	52	1.04	0.54	0.02	0.62	0.04
Ronaldo	176k	0.18	8	0.28	6.32	6	0.30	0.10	40	1.19	0.21	100	2.74	0.40	75	1.72	0.60	0.02	0.69	0.04
Isis	187k	0.16	4	0.18	6.25	3	0.17	0.09	15	0.45	0.22	100	2.66	0.31	70	1.39	1.12	0.02	0.65	0.04
Blade Smooth	195k	0.15	4	0.17	5.87	3	0.17	0.09	18	0.60	0.18	100	2.42	0.40	42	0.94	0.76	0.02	0.69	0.04
Max Planck	199k	0.16	7	0.26	5.98	4	0.19	0.08	19	0.55	0.17	100	2.20	0.30	34	0.69	1.94	0.03	0.67	0.05
Vase-Lion	200k	0.17	4	0.20	6.54	4	0.22	0.12	10	0.43	0.21	100	2.68	0.33	36	0.88	0.40	0.02	0.79	0.04
Duck	204k	0.16	8	0.26	5.91	7	0.27	0.08	16	0.45	0.18	100	2.47	0.37	69	1.39	1.68	0.03	0.64	0.05
Mouse	214k	0.19	6	0.23	6.33	4	0.20	0.10	21	0.61	0.20	100	2.60	0.39	60	1.28	1.65	0.03	0.70	0.05
Wolf Skull	228k	0.23	7	0.33	8.07	5	0.33	0.13	38	1.52	0.27	100	3.47	0.53	53	1.59	0.39	0.02	0.92	0.05
Moses	258k	0.42	12	0.55	8.72	5	0.35	0.30	100	4.30	0.27	100	3.64	0.55	100	3.33	0.90	0.02	1.04	0.05
Rockerarm	271k	0.27	3	0.20	9.20	3	0.27	0.16	5	0.34	0.28	100	2.90	0.52	22	0.53	1.79	0.03	1.01	0.06
Pulley2	293k	0.24	5	0.32	8.87	4	0.30	0.14	24	1.04	0.28	100	3.67	0.58	38	1.19	3.03	0.04	1.07	0.07
Heraklion	350k	0.36	20	1.18	12.95	7	0.67	0.21	100	5.79	0.46	100	5.82	0.80	100	4.54	2.39	0.04	1.45	0.08
Julius Caesar	387k	0.30	11	0.58	12.10	17	1.09	0.16	28	1.54	0.39	100	4.89	0.77	70	2.79	5.07	0.06	1.29	0.09
Goyle	393k	0.33	2	0.21	11.44	2	0.26	0.21	12	0.73	0.39	100	4.75	0.79	25	0.97	8.43	0.07	1.32	0.10
Eros	476k	0.41	NaN	NaN	14.78	5	0.55	0.21	28	1.90	0.46	100	6.01	0.97	71	3.55	4.03	0.06	1.76	0.11
Roal	484k	0.47	5	0.54	16.52	4	0.59	0.37	37	3.27	0.53	100	6.86	0.84	53	3.08	1.34	0.04	2.08	0.09
Skeleton	494k	0.71	9	1.10	20.60	NaN	NaN	0.35	100	9.18	0.66	100	9.42	1.63	100	7.82	3.33	0.05	2.33	0.11
Bimba	502k	0.43	7	0.65	15.58	6	0.74	0.24	48	4.16	0.51	100	6.97	1.15	69	4.45	3.54	0.05	2.13	0.10
Oil Pump	570k	0.47	19	1.57	18.76	12	1.31	0.25	45	3.93	0.55	100	7.29	1.27	74	4.92	6.85	0.07	2.30	0.13
Antique Head	651k	0.53	4	0.51	20.07	4	0.60	0.28	14	1.22	0.64	100	8.48	1.37	67	4.33	15.02	0.11	2.43	0.17
Pulley	660k	0.54	19	1.81	21.54	16	1.88	0.28	53	5.23	0.63	100	8.46	1.49	58	4.31	12.94	0.10	2.68	0.16
Beard Man	691k	0.59	4	0.56	22.15	3	0.58	0.26	14	1.43	0.52	100	7.07	1.46	14	0.96	24.57	0.14	2.72	0.19
Red Circular Box	701k	0.64	6	0.74	22.97	6	0.97	0.34	66	6.67	0.72	100	8.98	1.51	66	5.01	17.76	0.11	2.84	0.17
Dancing Children	724k	0.69	9	1.10	23.21	8	1.25	0.41	39	4.54	0.68	100	9.32	1.23	100	8.70	6.18	0.09	2.78	0.17
John The Baptist	750k	0.73	69	7.74	29.90	10	1.76	0.47	81	10.56	0.86	100	10.53	1.86	100	9.77	6.43	0.08	3.08	0.17
Ramses	826k	0.79	7	1.21	28.90	5	1.18	0.47	40	6.03	0.91	100	11.82	2.08	49	5.40	6.24	0.09	3.60	0.18
Nicolo Da Uzzano	946k	0.76	7	1.02	29.88	6	1.16	0.40	33	4.15	0.85	100	12.08	2.05	82	8.06	17.75	0.16	3.53	0.24
Raptor	1m	0.89	NaN	NaN	32.27	NaN	NaN	0.43	NaN	NaN	1.07	100	13.98	1.83	100	12.14	0.79	0.00	3.77	0.20
Nefertiti	1m	0.89	4	0.94	34.22	4	1.18	0.56	65	11.69	1.09	100	14.74	2.58	46	6.14	9.15	0.10	4.54	0.22
Isidore Horse	1.1m	1.12	11	1.90	35.60	5	1.27	0.50	70	11.03	1.11	100	14.60	2.50	88	10.72	24.01	0.17	4.56	0.28
Horse Head	1.3m	2.42	44	10.76	54.11	10	4.52	1.00	100	24.52	2.12	100	24.57	4.25	100	20.97	12.99	0.13	6.55	0.30
Ram	1.3m	2.40	3	1.95	56.18	NaN	NaN	1.14	49	13.39	2.45	100	25.95	4.72	100	21.71	17.07	0.15	6.99	0.33
Murex Romosus	1.8m	2.32	6	2.85	73.05	5	3.32	0.99	63	20.79	2.38	100	29.83	5.08	58	15.28	40.06	0.26	9.13	0.44
XYZ Dragon	3.6m	3.24	9	5.32	121.97	7	5.32	1.57	55	28.95	3.16	100	43.75	9.14	75	30.54	77.62	0.69	15.88	0.94

Table 2: Comparison of our hierarchy construction and solver for data smoothing of a random function with smoothing coefficient $\alpha = 1 \times 10^{-3}$ and tolerance of 1×10^{-4}. The maximum number of iterations for iterative solvers is set to 100.

Model	#Vert	Gravo MG (Ours)			Liu et al. [2021]			Shi et al. [2006]			AMG-RS			AMG-SA			Eigen		Pardiso	
		Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Fact.	Subst.	Fact.	Subst.
Beetle	19k	0.01	27	0.06	0.55	12	0.04	0.01	24	0.07	0.03	75	0.22	0.04	92	0.18	0.02	0.00	0.06	0.00
Ogre	19k	0.01	6	0.02	0.54	7	0.03	0.01	8	0.02	0.03	12	0.03	0.04	16	0.03	0.02	0.00	0.06	0.00
Screwdriver	27k	0.02	4	0.03	0.75	4	0.03	0.01	7	0.03	0.03	14	0.05	0.04	12	0.03	0.05	0.00	0.08	0.00
Mumble	34k	0.03	9	0.04	1.00	6	0.04	0.02	7	0.04	0.04	29	0.12	0.05	46	0.13	0.04	0.00	0.11	0.01
Horse	48k	0.04	6	0.04	1.38	6	0.06	0.02	8	0.06	0.05	34	0.20	0.07	15	0.07	0.10	0.00	0.14	0.01
Laurent's Hand	50k	0.04	5	0.07	1.56	4	0.06	0.02	9	0.08	0.06	17	0.11	0.11	12	0.07	0.10	0.00	0.17	0.01
Dinosaur	56k	0.04	5	0.05	1.72	4	0.06	0.02	7	0.07	0.06	57	0.37	0.09	22	0.13	0.08	0.00	0.18	0.01
Heart	78k	0.06	16	0.19	2.92	26	0.34	0.04	16	0.22	0.08	7	0.08	0.13	60	0.58	0.17	0.01	0.27	0.01
Hannya Mask	83k	0.07	8	0.11	2.56	3	0.08	0.05	11	0.16	0.08	34	0.35	0.13	100	0.92	0.17	0.01	0.30	0.02
Trex	100k	0.10	7	0.13	3.40	4	0.14	0.04	12	0.21	0.13	100	1.40	0.18	23	0.26	0.20	0.01	0.38	0.02
The Thinker	110k	0.09	4	0.08	3.48	3	0.10	0.04	7	0.11	0.10	20	0.22	0.21	11	0.12	0.35	0.01	0.39	0.02
Egea	134k	0.11	5	0.14	4.40	4	0.14	0.06	7	0.17	0.15	35	0.63	0.22	12	0.16	0.90	0.02	0.50	0.03
Sappho's Head	140k	0.11	6	0.16	4.38	7	0.22	0.07	8	0.22	0.14	100	1.79	0.29	16	0.26	0.38	0.01	0.51	0.03
Human Torso	142k	0.15	4	0.15	4.88	4	0.17	0.06	10	0.24	0.16	97	1.77	0.25	12	0.19	0.74	0.02	0.57	0.04
Aim Dragon	152k	0.13	6	0.17	5.07	8	0.27	0.06	14	0.35	0.15	6	0.11	0.31	10	0.17	0.80	0.02	0.58	0.04
Armadillo	172k	0.15	9	0.27	5.88	7	0.28	0.08	20	0.57	0.20	15	0.36	0.35	34	0.68	0.54	0.02	0.62	0.04
Ronaldo	176k	0.18	7	0.26	6.35	5	0.26	0.10	13	0.44	0.20	64	1.75	0.40	38	0.88	0.62	0.02	0.69	0.04
Isis	187k	0.16	3	0.16	6.29	3	0.18	0.09	10	0.32	0.22	23	0.61	0.31	46	0.92	1.13	0.02	0.65	0.04
Blade Smooth	195k	0.15	4	0.17	5.85	3	0.17	0.09	8	0.30	0.18	21	0.51	0.40	16	0.36	0.76	0.02	0.69	0.04
Max Planck	199k	0.16	7	0.26	5.95	4	0.20	0.08	8	0.26	0.18	100	2.20	0.30	20	0.42	1.97	0.03	0.67	0.05
Vase-Lion	200k	0.17	3	0.17	6.34	2	0.15	0.12	3	0.17	0.20	12	0.32	0.41	11	0.27	0.37	0.02	0.76	0.04
Duck	204k	0.15	6	0.21	5.89	5	0.21	0.08	11	0.32	0.19	100	2.46	0.37	25	0.50	1.66	0.03	0.65	0.05
Mouse	214k	0.19	5	0.21	6.38	4	0.20	0.10	11	0.35	0.20	100	2.59	0.40	31	0.66	1.68	0.03	0.71	0.05
Wolf Skull	228k	0.23	5	0.26	8.01	4	0.29	0.13	13	0.57	0.26	44	1.50	0.51	32	0.94	0.38	0.02	0.91	0.04
Moses	258k	0.48	7	0.37	8.64	4	0.31	0.36	44	1.95	0.27	100	3.64	0.45	100	3.34	0.92	0.02	1.04	0.05
Rockerarm	271k	0.27	3	0.21	9.21	3	0.27	0.17	3	0.27	0.28	22	0.64	0.53	10	0.25	1.84	0.03	1.01	0.06
Pulley2	293k	0.24	4	0.29	8.81	4	0.29	0.14	10	0.49	0.28	96	3.49	0.69	23	0.72	3.07	0.04	1.07	0.07
Heraklion	350k	0.36	13	0.84	12.83	5	0.55	0.20	24	1.52	0.46	100	5.85	0.80	100	4.65	2.43	0.04	1.45	0.08
Julius Caesar	387k	0.30	7	0.41	11.61	9	0.63	0.16	12	0.68	0.36	100	4.89	0.75	36	1.44	5.22	0.06	1.28	0.09
Goyle	393k	0.32	2	0.20	11.39	2	0.26	0.21	4	0.32	0.37	14	0.67	0.79	11	0.43	8.43	0.07	1.32	0.10
Eros	476k	0.41	7	0.55	14.80	5	0.55	0.21	11	0.84	0.46	100	5.98	1.19	35	1.73	4.18	0.07	1.77	0.12
Roal	484k	0.47	4	0.47	16.50	4	0.59	0.38	12	1.20	0.53	64	4.40	1.08	19	1.10	1.31	0.04	2.08	0.09
Skeleton	494k	0.69	7	0.95	20.03	NaN	NaN	0.34	18	1.95	0.69	100	9.45	1.65	41	3.23	3.45	0.05	2.32	0.11
Bimba	502k	0.43	6	0.58	15.73	5	0.65	0.24	14	1.34	0.52	100	7.05	0.89	35	2.28	3.54	0.05	2.13	0.10
Oil Pump	570k	0.47	18	1.51	18.79	11	1.22	0.25	30	2.67	0.55	90	6.62	1.30	46	3.10	7.13	0.07	2.36	0.13
Antique Head	651k	0.53	4	0.51	20.10	4	0.60	0.28	9	0.84	0.64	7	0.60	1.37	44	2.82	14.65	0.11	2.42	0.18
Pulley	660k	0.54	18	1.70	21.62	14	1.66	0.28	27	2.77	0.62	100	8.47	1.48	46	3.43	12.80	0.10	2.68	0.16
Beard Man	691k	0.58	4	0.56	22.44	3	0.58	0.26	7	0.81	0.52	5	0.36	1.78	6	0.42	24.65	0.14	2.71	0.19
Red Circular Box	701k	0.64	6	0.74	22.89	6	0.96	0.33	15	1.69	0.72	100	8.94	1.51	44	3.32	17.84	0.11	2.83	0.17
Dancing Children	724k	0.69	7	0.89	23.09	9	1.35	0.40	21	2.54	0.68	100	9.27	1.97	58	4.98	6.34	0.09	2.77	0.17
John The Baptist	750k	0.72	14	1.78	29.71	6	1.19	0.47	14	2.09	0.85	100	10.45	1.78	47	4.59	6.16	0.08	3.06	0.17
Ramses	826k	0.80	5	0.96	29.00	5	1.19	0.47	15	2.46	0.90	100	11.58	2.10	22	2.39	5.97	0.08	3.58	0.18
Nicolo Da Uzzano	946k	0.77	7	1.02	30.04	5	1.02	0.46	12	1.66	0.85	100	12.03	2.01	33	3.24	19.50	0.16	3.65	0.25
Raptor	1m	0.90	36	5.22	32.22	NaN	NaN	0.43	52	8.35	1.07	100	13.97	2.40	92	11.14	4.61	0.11	4.24	0.23
Nefertiti	1m	0.89	4	0.94	34.34	4	1.18	0.57	14	2.79	1.09	57	8.41	2.55	14	1.87	9.23	0.11	4.56	0.22
Isidore Horse	1.1m	0.94	10	1.65	35.42	5	1.28	0.50	24	3.98	1.10	100	14.46	2.47	74	8.89	23.25	0.17	4.54	0.28
Horse Head	1.3m	2.36	20	5.71	53.36	11	4.80	1.00	74	19.51	2.18	100	26.10	4.43	100	22.69	12.88	0.14	6.55	0.31
Ram	1.3m	2.72	3	2.18	57.16	NaN	NaN	1.21	5	2.53	2.32	83	21.68	4.68	25	5.49	17.59	0.15	7.05	0.33
Murex Romosus	1.8m	2.30	5	2.56	72.19	4	2.87	1.01	26	8.99	2.32	100	29.59	5.16	24	6.30	38.34	0.25	9.06	0.43
XYZ Dragon	3.6m	3.30	9	5.35	123.64	7	5.29	1.56	18	10.27	3.25	100	43.72	8.84	67	27.36	79.75	0.74	16.09	0.96

Table 3: Comparison of our hierarchy construction and solver for data smoothing of a random function with smoothing coefficient $\alpha = 1 \times 10^{-3}$ and tolerance of 1×10^{-4} on non-manifold meshes and point clouds. The maximum number of iterations for iterative solvers is set to 100.

| Model	#Vert	Gravo MG (Ours)			Shi et al. [2006]			AMG-RS			AMG-SA			Eigen		Pardiso	
		Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Hier	#It	Solve	Fact.	Subst.	Fact.	Subst.
NON-MANIFOLD TRIANGULAR MESHES																	
Lakoon	188k	0.16	5	0.20	0.09	26	0.88	0.20	100	2.76	0.40	17	0.39	0.43	0.02	0.71	0.04
Indonesian Statue	294k	0.26	7	0.42	0.17	7	0.44	0.30	37	1.45	0.63	100	3.53	0.90	0.03	1.18	0.06
Beethoven	383k	0.45	3	0.42	0.22	6	0.56	0.52	5	0.33	0.94	14	0.75	2.45	0.04	1.66	0.09
Bayon Lion	749k	1.44	4	1.20	0.71	8	1.67	1.37	7	1.07	2.45	10	1.28	6.26	0.08	3.75	0.18
Helmet Moustache	941k	2.04	5	2.07	0.74	22	4.97	2.10	15	2.97	3.44	17	2.79	24.99	0.14	5.56	0.26
Zeus	1.3m	2.51	7	2.69	1.20	14	4.52	2.49	29	7.24	4.19	100	20.66	30.96	0.19	7.26	0.35
Alfred Jacquemart	1.4m	3.26	4	3.33	1.68	13	6.16	3.61	7	2.44	5.24	28	8.24	9.08	0.14	8.03	0.35
POINT CLOUDS																	
Oil Pump	103k	0.07	4	0.07	0.04	6	0.11	0.10	7	0.09	0.19	12	0.13	0.17	0.01	0.30	0.02
Caesar Merged	388k	0.29	4	0.30	0.17	7	0.52	0.41	7	0.39	0.83	31	1.40	4.81	0.06	1.50	0.10
Truck	1.2m	0.96	6	1.39	0.68	9	2.14	1.27	12	2.30	3.69	46	6.88	5.81	0.15	5.24	0.29
Ignatius	1.4m	1.25	6	1.67	0.78	15	4.14	1.58	30	6.44	4.43	100	17.64	8.89	0.18	6.11	0.35
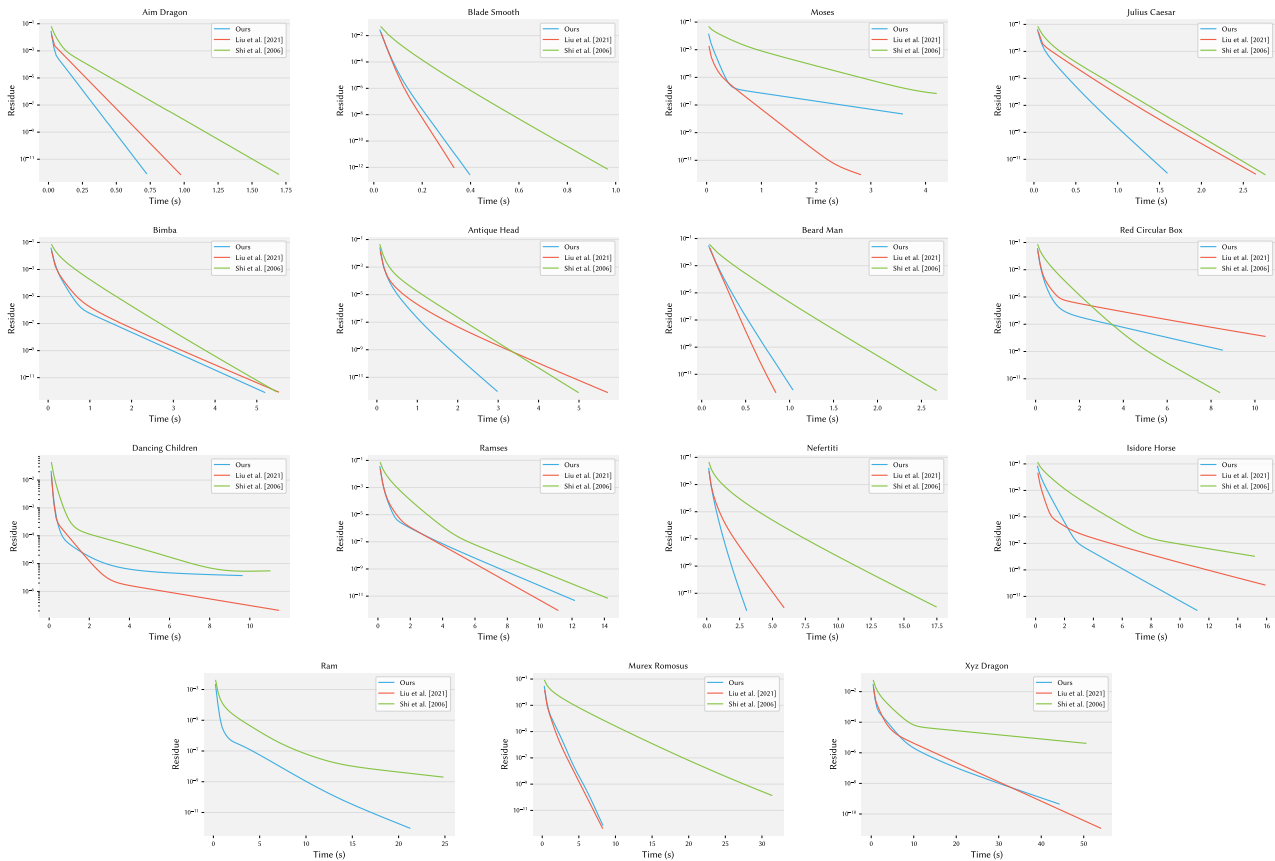
Figure 1: Convergence plots showing time on the x-axis for smoothing with $\alpha = 1 \times 10^{-3}$.
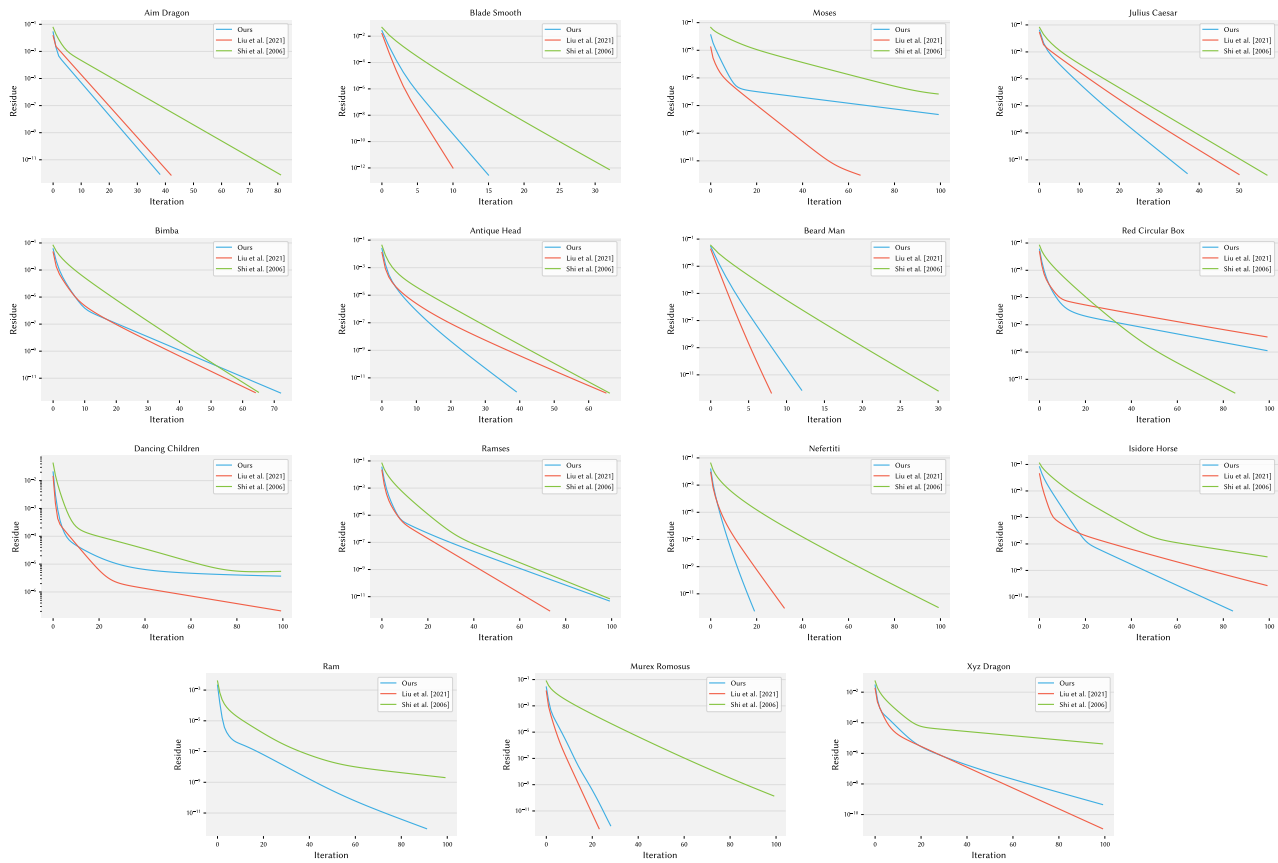
Figure 2: Convergence plots showing iterations on the x-axis for smoothing with $\alpha = 1 \times 10^{-3}$.